1. Variational derivative of a functional in calculus of variations is conceptually similar to which one of the following in calculus of functions?

(a) Directional derivative

(b) Partial derivative

(c) Fractional derivative

(d) None of the above

2. Which one of the following is not a functional?

(a) A ratio of two integrals

(b) An integral that contains an integral in its integrand

(c) Exponential of a function

(d) Maximum value of a function

3. Choose the correct alternative by examining the assertions and reasoning:

Assertions: (i) The transverse deformations of an extensible straight string held taut between two points under an arbitrary transverse load belong to a Sobolev space; (ii) A function space for which a Sobolev norm is finite includes discontinuous functions.

Reasoning: An extensible straight string held taut between two points can transversely deform to a discontinuous function under a particular transverse load applied on it.

(a) Both assertions are correct and the reasoning is correct.

(b) Both assertions are correct but the reasoning is not correct.

(c) Assertion (i) is correct but not the second.

(d) Assertion (ii) is correct but not the first.

4. Choose the correct statement among the following.

(a) Every converging sequence in a Banach space need not have its limit within the space.

(b) A norm is useful to compare the "distance" between a function and the additive identity of the function space to which the function belongs.

(c) Inner product of two functions is not a functional.

(d) A vector space cannot have more than one norm defined for it.

5. If $J = \dfrac{\int_0^2 y^2 dx}{\int_0^2 yy'\,dx}$, Gateaux variation of J with respect to $y(x)$, i.e., $\delta J(y,h)$, is given by

(a) $\dfrac{\int_0^2 (2y)\,hdx}{\int_0^2 (y'h + yh')\,dx}$

(b) $\dfrac{\left(\int_0^2 yy'dx\right)\left\{\int_0^2 (2y)\,hdx\right\} - \left(\int_0^2 y^2 dx\right)\left\{\int_0^2 (y'h + yh')\,dx\right\}}{\left(\int_0^2 yy'\,dx\right)^2}$

(c) $-\dfrac{\left(\int_0^2 2yhdx\right)\left\{\int_0^2 (2y'h + yh')\,hdx\right\}}{\left(\int_0^2 yy'\,dx\right)^2}$

(d) $\int_0^2 \left[\dfrac{\partial}{\partial y}\left(\dfrac{y^2}{yy'}\right) - \dfrac{d}{dx}\left\{\dfrac{\partial}{\partial y'}\left(\dfrac{y^2}{yy'}\right)\right\}\right] hdx$

6. Given that $J = \int_0^L \left(a\dot{y}^2 + y^3\right) dt$, identify the variational derivative of J with respect to $y(t)$ from the following alternatives.

(a) $3y^2 - 2a\ddot{y}$

(b) $3y^2 + 2a\ddot{y}$

(c) $-3y^2 + 2a\ddot{y}$

(d) $-3y^2 - 2a\ddot{y}$

7. Which one of the statements is not true in view of the properties of a Banach space?

(a) Banach space has a norm defined for it.

(b) Lebesgue space is a Banach space.

(c) A Banach space can have the limit of its converging sequence outside itself.

(d) Banach space satisfies the properties of a vector space.

8. Choose the correct expression from the following three options given as possible first variation of the functional, $J = \int_a^b \left(y'' + y'^2 + yy' \right) dx$.

(i) $J = \int_a^b \left\{ y' - \frac{d}{dx}(2y' + y) \right\} h\, dx$

(ii) $J = \int_a^b \left\{ y' - \frac{d}{dx}(2y' + y) \right\} dx + (2y' + y) h\Big|_a^b + h'\Big|_a^b$

(iii) $J = \int_a^b \left\{ y' - \frac{d}{dx}(2y' + y) \right\} h\, dx + (2y' + y) h\Big|_a^b + h'\Big|_a^b$

(a) Only (i) is correct

(b) Only (ii) is correct.

(c) Only (iii) is correct.

(d) Only (i) and (iii) are correct.

9. Choose the correct alternative by examining the two assertions and reasoning:

Assertions: (i) A Banach space is a complete normed vector space. (ii) In calculus of variations, the space in which we search for a minimizing function should be a Banach space.

Reasoning: In iterative optimization involving functionals, as we search from an initial guess and generate a sequence of functions, which ultimately should converge to the minimizing function that belongs to the space function space.

(a) Both assertions are correct and the reasoning is correct.

(b) Both assertions are correct but the reasoning is not correct.

(c) Assertion (i) is correct but not the second.

(d) Assertion (ii) is correct but not the first.

10. If $\mathbf{A} = (x_1, \mathrm{y}_1)$ and $\mathbf{B} = (x_2, \mathrm{y}_2)$ are two vectors in a 2D space, which one of the metrics satisfy the four properties of a metric?

(i) $d(\mathbf{A},\mathbf{B}) = \begin{cases} 1 \ if \ \ \mathbf{A} \neq \mathbf{B} \\ 0 \ \ if \ \ \mathbf{A} = \mathbf{B} \end{cases}$

(ii) $d(\mathbf{A},\mathbf{B}) = \sqrt{(x_2 - x_1)^2 + (\mathrm{y}_2 - y_1)^2}$

(iii) $d(\mathbf{A},\mathbf{B}) = |x_2 - x_1| + |\mathrm{y}_2 - y_1|$

(a) Only (i) and (ii)

(b) Only (ii)

(c) Only (ii) and (iii)

(d) All of the above

11. Inner product of two functions is defined as the integral of their product over the domain. Then, what is the inner product of sin(x) and cos(x) from 0 to pi?

(a) 1

(b) 2

(c) -1

(d) 0

12. Which one of the following is not essential to define a vectors space?

(a) A scalar field

(b) Vector addition operation

(c) The operation of a scalar multiplying a vector

(d) A norm

13. A Fréchet differential is…

(a) Linear

(b) Nonlinear

(c) Discontinuous

(d) Not equal to Gâteaux variation

14. If the potential energy of an axially loaded bar, $PE = \int_0^L \left(EAu'^2 - pu \right) dx$, is minimized with respect to axial deformation, $u(x)$, then the possible boundary conditions are:

(a) Variation of u is zero.

(b) u is zero.

(c) u' is zero.

(d) All of the above.

15. Given that $J = \int_0^p \left(y'^2 + y^2 y'^2 + yy'' \right) dx$ is minimized, identify the Euler-Lagrange equation:

(a) $yy'^2 + y^2 \left(y'' \right) = 0$

(b) $y'' + 2\left(y' \right)'' = 0$

(c) $y'' - 2\left(y'' \right)'' = 0$

(d) $y'' - 2y' + 2\left(y'' \right)'' = 0$